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ABSTRACT

A detailed gravimetric geoid of the United States, North Atlantic, and Eurasia, which was computed from a combination of satellite-derived and surface gravity data, is presented herein. The precision of this detailed geoid is ±2 to ±3 m in the continents but may be in the range of 5 to 7 m in those areas where data are sparse. Comparisons of the detailed gravimetric geoid with results of Rapp, Fischer, and Rice for the United States, Bomford in Europe, and Heiskanen and Fischer in India are presented. Comparisons are also presented with geoid heights from satellite solutions for geocentric station coordinates in North America, the Caribbean, and Europe.

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CONTENTS

A DETAILED GRAVIMETRIC GEOID FROM NORTH AMERICA TO EURASIA

INTRODUCTION

The objective of this work was to compute a detailed gravimetric geoid for the North Atlantic and Eurasia comparable to the detailed geoid computation of the United States that was presented at the Third International Symposium on Satellite Geodesy. This goal was achieved. Furthermore, the detailed geoid of the United States was recomputed when it became evident that data collected for the North Atlantic were of better quality than the data previously available for the eastern section of the detailed geoid computation for the United States. In this process of recomputation, a small computational error in the original geoid computation was corrected.

The detailed gravimetric geoid for the United States was shown to possess an accuracy of ±2 m rms. This accuracy was retained in the recomputation and it was also achieved in Eurasia when values of the detailed gravimetric geoid for Eurasia were compared with astrogeodetic geoid values of Bomford (1971), Heiskanen's Columbus geoid (1957), and Fischer's astrogeodetic geoid (1968).

No external standard is available to compare the accuracy of the detailed geoid in the North Atlantic because no other detailed geoid of this area has been published. However, based on the results obtained in the United States and western Europe and the quality of the data available in the North Atlantic, the accuracy may be expected to range from ±2 m rms where there are substantial data to ±5 to ±7 m where data are less dense and accurate.

This detailed gravimetric geoid extending from 122° W to 105° E longitude, covering the United States, the North Atlantic, and Eurasia, is believed to represent the first detailed gravimetric geoid of this substantial area ever published in the open literature.

METHOD OF COMPUTATION

The method of computation of the combination gravimetric geoid is described in great detail by Strange et al. (1971). The geoidal undulation of Earth at any point P is computed using the well-known Stokes' formula:

$$N(\phi, \lambda) = \frac{R}{4\pi G} \int_{\lambda'=0}^{2\pi} \int_{\phi'=-\pi/2}^{\pi/2} \Delta g_T(\phi', \lambda') S(\Psi) \cos \phi' \, d\phi' \, d\lambda'$$

where

ϕ, λ = the latitude and longitude, respectively, of the computation point;

ϕ', λ' = the latitude and longitude, respectively, of the variable integration point;

$N(\phi, \lambda)$ = geoid undulation at (ϕ, λ);

R = mean radius of Earth;

G = product of the universal gravitational constant and the mass of Earth;

$\Delta g_T(\phi', \lambda')$ = free air gravity anomaly at the variable point (ϕ', λ');

$S(\Psi)$ = value of Stokes' function.

The combined geoid undulation results from using local surface gravity for the near area (10° around the computational point) and satellite gravity in the far area (>10° from the computational point).

The final detailed gravimetric geoid is presented in Figure 1 (see pocket on inside of back cover). The geoid is referenced to an ellipsoid with a flattening given by $f = 1/298.255$.

DETAILED GRAVIMETRIC GEOID FOR THE UNITED STATES

Absolute Adjustment

Tables 1 through 8 present comparisons of the geoid heights computed in the present analysis with those computed using the geocentric x, y, z positions for stations in North America derived from satellite data by various investigators. In those cases where North American datum (NAD) positions were published rather than geocentric, these were converted to the geocentric system using $\Delta x = -36$, $\Delta y = 170$, and $\Delta z = 191$ for Air Force Cambridge Research Laboratories (AFCRL) data; $\Delta x = -32$, $\Delta y = 159$, and $\Delta z = 171$ for Ohio State data; and $\Delta x = -30$, $\Delta y = 152$, and $\Delta z = 176$ for GSFC short-arc and Aeronautical Charting and Information Center (ACIC) data.

The geoid heights are each given with reference to two ellipsoids. For example, in Table 1 the geoid heights of the GSFC long-arc solution[1] within the North American Continent using semimajor axes of 6378.155 and 6378.126 km and retaining the 1/298.255 flattening are compared with the gravimetric geoid heights. The reason for this dual set of semimajor axes is summarized below.

When the detailed gravimetric geoid heights are compared to the geoid heights of the GSFC long-arc solution referenced to a semimajor axis of 6378.155 km (Table 1), a systematic difference of 29 m was noted. One possible reason for the difference was a known uncertainty in the value of sea-level equipotential W_0. Rather than recomputing the gravimetric geoid using an adjusted value for W_0 to remove this scale difference, an equivalent adjustment was made to the value of a_e characterizing the reference ellipsoid. This consisted in subtracting 29 m from the value originally used, resulting in a value of 6378.126 km.

[1]See Table 1, Footnote b, page 7.

The meaning of this is that for the gravimetric geoid heights to be compatible with the GSFC long-arc geocentric station positions, they must be considered as heights above an ellipsoid with a semimajor axis of 6378.126 km.

Tables 2 through 8 represent similar comparisons between the results of various investigators and the gravimetric geoid. As may be seen, the value of the semimajor axis of the reference ellipsoid to which the gravimetric geoid heights must be compared to agree with the geocentric station positions varies from one investigator to another. This implies that small systematic differences exist between the radial positions of stations derived by different investigators. In general, the mean difference is less than 10 m.

Comparative Evaluation

To evaluate the precision of the detailed geoid for the United States, a number of comparisons were made. The first comparison was made with the computations of Rapp (1967) in a portion of western United States. Figure 2 presents a plot of these comparisons. In this comparison 6 m were subtracted from the results of Rapp so that a comparison of relative shape is effected. As is clearly seen, the agreement as to the relative geoid shape is exceptional considering that Rapp used different procedures and data.

Another source of comparison was the astrogeodetic geoid data of Rice (1970). Before any comparisons could be made, Rice's data[2] were transformed from NAD to the geocentric coordinates using various transformation sets. Table 9 presents the differences between Rice's astrogeodetic geoid and the gravimetric geoid using four different sets of translational elements and removing the mean differences. In all cases, the rms differences are on the order of 2 m or less. When using the translational values of Fischer (1968), the largest disagreement with Rice's astrogeodetic data appears in the Florida area. On the other hand, Fischer's results agree well with Rice's data in the Louisiana, Mississippi Gulf Coast area. The opposite situation results when the transformation constants of other investigators are used. A possible explanation for this fact could be that in determining astrogeodetic transformation values, Fischer may have given greater weight to the mid-Gulf Coast area where a greater number of astrogeodetic deflection stations exist.

A final comparison was made using transformation sets 1 and 2 from Table 9 to transform geoid profile data taken from the map of Fischer (1967) to the geocentric system. These comparisons at latitudes 40° N and 45° N after removing a systematic difference of 18 m are presented in Figures 3 and 4. Again the relative agreement is within the ±2-m range.

An important question that can be studied from the results obtained in the United States is the question of possible rotation in the North American datum. We see in Table 9 and Figures 3 and 4 random variations but no indications of any substantial rotation. If any datum rotation does exist, it must be very small (i.e., less than 0.2 arcsec).

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[2]D. A. Rice, 1970, "Adjustment of Geoidal Sections in the United States, 1927 American Datum," personal communication.

DETAILED GRAVIMETRIC GEOID FOR THE NORTH ATLANTIC

The major source of the surface gravity data in the Caribbean area was Bowin[3] of Woods Hole Oceanographic Institution. Other primary data sources for the Atlantic were Talwani[4] of Lamont-Doherty Geological Observatory and Strang van Hees (1970), and a set of 1°-by-1° mean anomalies obtained from ACIC.

Absolute Adjustment

Tables 10 through 15 present comparisons of geoid heights computed in the present analysis and geoid heights computed using satellite-derived geocentric x, y, and z positions for tracking stations in the North Atlantic. The same systematic differences were used here as in Tables 1 through 8.

In the North Atlantic only GSFC long-arc,[5] GSFC short-arc, and AFCRL data had mean differences of less than 10 m between their geoid heights and the gravimetric geoid heights. The substantial error for Grand Turk in the Wallops Island C-band solution was to be expected because the investigators later found errors in the data used in the solution for this station. The majority of the investigators had large disagreements for station 7076, Jamaica, but agreed reasonably well with one another. The reason for this disagreement is not clear.

Comparative Evaluation

As mentioned earlier, there was no external standard available to compare the accuracy of the detailed geoid for the North Atlantic because no other detailed geoid for the North Atlantic has been published. However, it is encouraging to find that the dynamically derived station positions do not systematically disagree with the gravimetric results except at Jamaica. If we compare a profile of the detailed gravimetric geoid with that derived by satellite at longitude 67° W (Figure 5), we notice that the Puerto Rico trench stands out quite clearly. The maximum relief in the geoid across the trench measures approximately 10 m in dip, which is close to what Von Arx (1966) measured on his study of the ocean surface over the Puerto Rico trench.

Another profile at latitude 17° N (Figure 6) again shows the great details evident in the gravimetric geoid of the Caribbean. In contrasting the satellite geoid and the detailed geoid for the Caribbean, many features are evident. First the extent of the trench is more clearly defined and so are the highs associated with the Lesser Antilles Islands chain. The lows extending toward the tip of South America are also apparent. The steep gradient evident all along the trench and across the North Atlantic (Figure 7) can very well serve as a test area where the altimeter on the future GEOS C satellite could be calibrated.

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[3] Carl Bowin, 1971, personal communication.
[4] M. Talwani, 1971, personal communication.
[5] See Table 1, Footnote b, page 7.

From Figure 1(a) it is seen that the steep gradient obscures the effect of the mid-Atlantic ridge on the trend of the geoid. This steep gradient extends all the way from -60 m in the Caribbean to +60 m in the Azores, a difference of 120 m in the North Atlantic.

DETAILED GRAVIMETRIC GEOID FOR EURASIA

The primary sources of the surface data used to derive the Eurasian part of the geoid map presented in Figure 1(b) were Tengström (1965), Arnold (1964), Strang van Hees (1970), and ACIC.[6]

Absolute Adjustment

Tables 16 and 17 present comparisons of the gravimetric geoid heights of Figure 1 similar to those carried out for North America and the Atlantic-Caribbean area. In this case the results of only two investigators were available. The systematic difference used for the GSFC long-arc results are the same as for the other areas investigated; however, in Table 17 when SAO station positions are compared with gravity geoid heights, the SAO geoid heights are each given with reference to three ellipsoids with semimajor axes of 6378.155, 6378.136, and 6378.115 km.

When the detailed gravimetric geoid heights at station sites in the United States and the North Atlantic were compared to SAO geoid heights referenced to the 6378.155-km ellipsoid, a systematic difference of 19 m was noted; however, when the same procedure was applied to SAO station sites in Europe, a systematic difference of 40 m was noted, which clearly indicates a scale error (systematic radial position error). This error was discussed with Lambeck[7] in Moscow in August 1971. Lambeck then indicated that he had found an error for the European solution because of erroneous geodetic coordinates for station 9091 in Greece.

The mean difference between the values of the SAO stations in Europe, after correction for a mean error of 21 m, and the detailed gravimetric geoid is 8 m as compared with a mean difference of 7 m for the GSFC station positions in Europe.

Two stations show substantial disagreement: 9091 Dionysos, Greece, and 9432 Uzhgorod, U.S.S.R. A possible source of the disagreement is uncertainty in the mean sea-level heights used in the computations.

Comparative Evaluation

To evaluate the accuracy of the final geoid for Eurasia, a comparison was made with Bomford's (1971) astrogeodetic geoid map of Europe, Heiskanen's (1957) Columbus geoid, and Fischer's (1968) astrogeodetic geoid of India. Bomford's astrogeodetic and Heiskanen's gravimetric geoid values were first transformed from the European datum to the geocentric system. The transformation sets used were those of GSFC, where $\Delta x = 89$, $\Delta y = 120$, and $\Delta z = 118$. It should be noted that these are mean translation values that do not incorporate

[6]ACIC, 1971, "1° × 1° Mean Free-Air Gravity Anomalies," private communication.
[7]K. Lambeck, 1971, personal communication.

the tilt of the European datum with respect to the geocentric system. The comparisons were made along profile lines at latitudes 44°, 48°, and 52° N. (See Figures 8 through 10.) The comparisons of the transformed astrogeodetic data and the gravimetric geoid along these east-west profiles showed a definite rotation in the European datum. This rotation was on the average equal to about 1.7 arc seconds. Such a rotation would appear reasonable because, as indicated by the gravimetric geoid, there is a systematic east-west tilt to the geoid across Europe. Allowing for this rotation in the datum, the relative agreement between Bomford's astrogeodetic geoid and the gravimetric geoid is within the ±2-m range.

The comparison made between Heiskanen's Columbus geoid and the gravimetric geoid along profiles at latitudes 35°, 40°, and 45° N (Figures 11 to 13) indicated an east-west rotation similar to the one derived when comparing the gravimetric geoid with Bomford's astrogeodetic geoid.

Another comparison was made with Fischer's astrogeodetic geoid of India along profiles at latitudes 24° and 28° N (Figures 14 and 15). The geoid values of Fischer had to be converted from the modified Mercury datum to the SAO C-6 system before a comparison could be made. After removal of a systematic difference of 9 m the relative agreement was again within a ±2-m range.

PROFILE ACROSS LONGITUDE 120° W TO 40° E AT LATITUDE 38° N

In Figure 7 a profile was drawn across approximately one-half the globe at latitude 38° N comparing the detailed gravimetric geoid and the SAO '69 satellite geoid. Several conclusions result from study of this profile:

(1) There is a steep gradient existing in the North Atlantic.

(2) There is no indication on the profile of any major tilt in North America.

(3) There is a tilt in the geoid of Europe.

(4) In general, the gravimetric geoid differs at most by approximately 10 m from the satellite geoid.

CONCLUSIONS

The detailed gravimetric geoid presented here has a precision of ±2 m.

One question that might have been answered in this study concerns the possible existence of a rotation in different major datums. From this study there seems to be no conclusive evidence of a rotation in the North American datum but a rotation, which is prominent along the east-west profile, does clearly exist in the European datum.

Another study conducted on this subject was the computation of detailed geoids for the United States using the APL 3-5 model and the SAO '69 model truncated at (8, 8). In both cases, the mean difference for profiles taken in the United States was on the order of ±2 m. This indicates that when sufficient data exist, results are similar no matter what satellite gravity model is used.

ACKNOWLEDGMENTS

Data preparation and computer runs were performed by Donald Smith and Michael Nichols.

Table 1—GSFC long-arc solution/gravimetric geoid comparisons.

(1) Station No.[a]	(2) GSFC Long-Arc Geoid[b] Height[c] (m)	(3) Adjusted GSFC Long-Arc Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
1021	-59	-30	-26	-4
1022	-47	-18	-18	0
1030	-53	-24	-27	3
1034	-49	-20	-18	-2
1042	-59	-30	-22	-8
7036	-52	-23	-12	-11
7037	-60	-31	-24	-7
7045	-44	-15	-13	-2
7050	-52	-23	-26	3
7072	-52	-23	-24	1
7075	-61	-32	-31	-1

[a]See appendix for station locations.
[b]Computed using data taken from J. G. Marsh, B. C. Douglas, and S. M. Klosko, 1971, "A Unified Set of Tracking Station Coordinates Derived From Geodetic Satellite Tracking Data." NASA GSFC document X-553-71-370.
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.126 km.

Table 2—SAO/gravimetric geoid comparisons.

(1) Station No.[a]	(2) SAO Geoid[b] Height[c] (m)	(3) Adjusted SAO Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
1021	-46	-27	-27	0
1034	-36	-17	-18	1
1042	-53	-34	-22	-12
7037	-45	-26	-24	-2
7045	-24	-5	-13	8
7050	-46	-27	-26	-1
7075	-56	-37	-31	-6
9001	-40	-21	-18	-3
9010	-44	-25	-24	-1
9021	-46	-27	-22	-5
9050	-50	-31	-20	-11
9113	-42	-23	-28	5

[a]See appendix for station locations.
[b]Computed using data taken from Gaposchkin and Lambeck (1970).
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.136 km.

Table 3—Naval Weapons Laboratory (NWL) gravimetric geoid comparisons.

(1) Station No.[a]	(2) NWL Geoid[b] Height[c] (m)	(3) Adjusted NWL Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
2	-24	0	-15	15
103	-46	-22	-21	-1
111	-53	-29	-25	-4
200	-62	-38	-31	-7
400	-46	-22	-17	-5
710	-46	-22	-27	5
711	-34	-10	-19	9
720	-48	-24	-32	8
734	-42	-18	-22	4
735	-45	-21	-24	3
736	-57	-33	-18	-15
737	-59	-35	-27	-8
738	-38	-14	-19	5
741	-40	-16	-18	2
742	-54	-30	-25	-5
745	-38	-14	-9	-5
748	-47	-23	-18	-5

[a]See appendix for station locations.
[b]Computed using data taken from Anderle and Smith (1967).
[c]Referenced to an ellipsoid with semimajor axis = 6378.166 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.142 km.

Table 4—GSFC short-arc solution/gravimetric geoid comparisons.

(1) Station No.[a]	(2) GSFC Short-Arc Geoid[b] Height[c] (m)	(3) Adjusted GSFC Short-Arc Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
1021	-37	-13	-26	13
1022	-47	-23	-18	-5
1030	-40	-16	-27	11
1034	-40	-16	-18	2
1042	-53	-29	-22	-7
7037	-47	-23	-24	1
7045	-28	-4	-13	9
7050	-49	-25	-26	1
7072	-51	-27	-24	-3

[a]See appendix for station locations.
[b]Computed using data taken from Loveless et al. (1970).
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.131 km.

Table 5–Ohio State geometric solution/gravimetric geoid comparisons.

(1) Station No.[a]	(2) Ohio State Geoid[b] Height[c] (m)	(3) Adjusted Ohio State Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
1021	-71	-33	-26	-7
1022	-55	-17	-18	1
1030	-57	-19	-27	8
1034	-45	-7	-18	11
1042	-63	-25	-22	-3
3402	-68	-30	-18	-12
3648	-67	-29	-24	-5
3657	-68	-30	-26	-4
5861	-65	-27	-22	-5
7037	-52	-14	-24	10
7045	-35	3	-13	16
7050	-79	-41	-26	-15
7072	-61	-23	-24	1
7075	-62	-24	-31	7

[a]See appendix for station locations.
[b]Computed using data taken from Mueller et al. (1970).
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.117 km.

Table 6–AFCRL/gravimetric geoid comparisons.

(1) Station No.[a]	(2) AFCRL Geoid[b] Height[c] (m)	(3) Adjusted AFCRL Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
1021	-53	-32	-26	-6
1022	-29	-8	-18	10
1034	-37	-16	-18	2
3402	-46	-25	-18	-7
3648	-48	-27	-24	-3
3657	-53	-32	-26	-6
5001	-56	-35	-25	-10
5333	-40	-19	-19	0
5861	-43	-22	-22	0
7037	-45	-24	-24	0
7045	-26	-5	-13	8
7050	-54	-33	-26	-7
7051	-44	-23	-22	-1
7072	-47	-26	-24	-2
7075	-48	-27	-31	4

[a]See appendix for station locations.
[b]Computed using data taken from Hadgigeorge (1970).
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.134 km.

Table 7—ACIC/gravimetric geoid comparisons.

(1) Station No.[a]	(2) ACIC Geoid[b] Height[c] (m)	(3) Adjusted ACIC Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
3402	-50	-28	-18	-10
3647	-48	-26	-17	-9
3648	-50	-28	-24	-4
3657	-53	-31	-26	-5
3861	-43	-21	-22	1
5333	-45	-23	-19	-4

[a]See appendix for station locations.
[b]Computed using data taken from ACIC (1969).
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.133 km.

Table 8—Wallops Island C-band/gravimetric geoid comparisons.

(1) Station No.[a]	(2) Wallops Island C-Band Geoid[b] Height[c] (m)	(3) Adjusted Wallops Island C-Band Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
4082	-46	-16	-23	7
4280	-39	-9	-32	23
4860	-63	-37	-27	-10
7050	-48	-18	-26	8

[a]See appendix for station locations.
[b]Computed using data taken from R. L. Brooks, 1970, "Tables of C-Band Radar Station Positions," personal communication.
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.125 km.

Table 9—Comparison of detailed gravimetric geoid and Rice's astrogeodetic geoid under varying assumptions for transforming astrogeodetic data.

Latitude, N	Longitude, W	1	2	3	4
34° 58′ 03″.0	120° 38′ 05″.5	2	-1	4	5
35 00 38.0	119 00 48.0	2	0	5	5
38 47 23.1	121 52 15.6	1	-2	3	4
35 02 36.1	106 30 24.1	1	1	3	3
32 13 14.7	106 29 41.6	4	3	5	6
32 00 00.6	103 16 07.2	0	0	2	2
30 59 40.0	098 05 50.5	-2	-1	0	0
30 36 26.5	091 23 18.1	-3	-1	-1	-1
29 38 10.8	091 06 49.3	-4	-2	-3	-2
30 59 25.5	089 34 29.5	-3	-1	-2	-1
28 29 28.6	080 33 35.6	2	4	2	3
30 36 53.3	081 42 14.8	1	4	2	3
39 28 18.9	076 05 15.2	-2	1	-2	0
34 59 44.0	076 59 11.7	-3	1	-2	0
33 28 42.4	091 00 08.5	-1	1	0	1
33 34 48.5	092 50 07.2	-1	1	0	1
34 56 47.0	093 24 18.3	-3	-2	-2	-1
37 38 08.4	094 35 46.8	-3	-1	-1	0
35 03 04.0	097 56 52.6	0	0	1	2
39 13 26.7	098 32 30.5	-1	-1	0	0
43 37 10.7	096 17 52.3	-1	0	0	0
35 06 16.2	103 19 55.0	1	1	2	3
34 56 32.8	096 24 55.3	0	1	1	2
44 43 46.0	105 25 50.7	2	1	2	3
36 47 44.2	103 11 48.5	0	0	1	2
38 50 40.6	102 48 46.8	-1	-1	1	1
48 06 18.6	102 21 09.7	-2	-2	-1	0
46 44 47.4	102 15 13.4	-2	-2	-1	0
45 12 45.7	102 09 14.1	0	-2	-1	0
46 21 53.1	108 59 07.3	3	2	4	5
31 03 07.3	102 56 05.8	0	0	1	2
41 30 41.9	097 37 23.4	-1	0	0	1
30 48 49.8	093 12 26.9	-5	-1	-5	-4
47 50 28.9	110 00 46.4	1	0	2	3

1 = Corrected difference between Rice's astrogeodetic geoid and the detailed gravimetric geoid using Marsh's (Marsh et al., see Table 1, Footnote b, page 7) translation values of $\Delta x = -25.1$, $\Delta y = 162.9$, and $\Delta z = 172.5$.

2 = Corrected difference between Rice's astrogeodetic geoid and the detailed gravimetric geoid using Fischer's (1968) translation values of $\Delta x = -18$, $\Delta y = 145$, and $\Delta z = 183$.

3 = Corrected difference between Rice's astrogeodetic geoid and the detailed gravimetric geoid using SAO Standard Earth '66 translation values of $\Delta x = -30$, $\Delta y = 152$, and $\Delta z = 176$ (Lundquist and Veis, 1967).

4 = Corrected difference between Rice's astrogeodetic geoid and the detailed gravimetric geoid using SAO's translation values of $\Delta x = -25.8$, $\Delta y = 168.1$, and $\Delta z = 167.0$ (K. Lambeck, 1971, personal communication).

Table 10—GSFC long-arc solution/gravimetric geoid comparisons.

(1) Station No.[a]	(2) GSFC Long-Arc Geoid[b] Height[c] (m)	(3) Adjusted GSFC Long-Arc Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
7039	-58	-29	-36	7
7040	-68	-39	-41	2
7076	-41	-12	-23	11

[a]See appendix for station locations.
[b]Computed using data taken from Marsh et al. (See Table 1, Footnote b, page 7.)
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.126 km.

Table 11—SAO/gravimetric geoid comparisons.

(1) Station No.[a]	(2) SAO Geoid[b] Height[c] (m)	(3) Adjusted SAO Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
7039	-40	-21	-36	15
7040	-58	-39	-41	2
7076	-14	5	-23	28

[a]See appendix for station locations.
[b]Computed using data taken from Gaposchkin and Lambeck (1970).
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.136 km.

Table 12—GSFC short-arc solution/gravimetric geoid comparisons.

(1) Station No.[a]	(2) GSFC Short-Arc Geoid[b] Height[c] (m)	(3) Adjusted GSFC Short-Arc Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
7039	-59	-35	-36	1
7040	-80	-56	-41	-15
7076	-51	-27	-23	-4

[a]See appendix for station locations.
[b]Computed using data taken from Loveless et al. (1970).
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.131 km.

Table 13—AFCRL/gravimetric geoid comparisons.

(1) Station No.[a]	(2) AFCRL Geoid[b] Height[c] (m)	(3) Adjusted AFCRL Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
3106	-63	-42	-39	-3
3405	-60	-39	-31	-8
7039	-51	-30	-36	6
7040	-61	-40	-41	1
7076	-24	-3	-23	20

[a]See appendix for station locations.
[b]Computed using data taken from Hadgigeorge (1970).
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.134 km.

Table 14—Ohio State geometric solution/gravimetric geoid comparisons.

(1) Station No.[a]	(2) Ohio State Geoid[b] Height[c] (m)	(3) Adjusted Ohio State Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
3106	-79	-41	-39	-2
3405	-96	-58	-31	-27
7039	-76	-38	-36	-2
7040	-74	-36	-41	5
7076	-41	-3	-23	20

[a]See appendix for station locations.
[b]Computed using data taken from Mueller et al. (1970).
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.117 km.

Table 15—Wallops Island C-band/gravimetric geoid comparisons.

(1) Station No.[a]	(2) Wallops Island C-Band Geoid[b] Height[c] (m)	(3) Adjusted Wallops Island C-Band Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
4061	-61	-31	-39	8
4081	-94	-64	-31	-33

[a]See appendix for station locations.
[b]Computed using data taken from R. L. Brooks, 1970, "Tables of C-Band Radar Station Positions," personal communication.
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.125 km.

Table 16–GSFC long-arc solution/gravimetric geoid comparisons.

(1) Station No.[a]	(2) GSFC Long-Arc Geoid[b] Height[c] (m)	(3) Adjusted GSFC Long-Arc Geoid Height[d] (m)	(4) Gravimetric Geoid Height (m)	(5) (3) - (4) (m)
1035	23	52	50	2
8009	21	50	47	3
8010	30	59	54	5
8015	33	62	55	7
8019	28	57	55	2
9004	30	59	55	4
9091	15	44	54	-10
9115	19	48	46	2
9431	-23	6	24	-18
9432	16	45	47	-2

[a]See appendix for station locations.
[b]Computed using data taken from Marsh et al. (See Table 1, Footnote b, page 7.)
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.126 km.

Table 17–SAO/gravimetric geoid comparisons.

(1) Station No.[a]	(2) SAO Geoid[b] Height[c] (m)	(3) Adjusted SAO Geoid Height[d] (m)	(4) Readjusted SAO Geoid Height[e] (m)	(5) Gravimetric Geoid Height (m)	(6) (3) - (5) (m)	(7) (4) - (5) (m)
8015	16	35	56	55	-20	1
8019	17	36	57	55	-19	2
9004	25	44	65	55	-11	10
9065	7	26	47	47	-21	0
9066	14	33	54	54	-21	0
9080	21	40	61	52	-12	9
9091	-12	7	28	54	-47	-26
9115	8	27	48	46	-19	2
9431	-27	-8	13	24	-32	-11
9432	-2	17	38	47	-30	-9

[a]See appendix for station locations.
[b]Computed using data taken from Gaposchkin and Lambeck (1970).
[c]Referenced to an ellipsoid with semimajor axis = 6378.155 km.
[d]Referenced to an ellipsoid with semimajor axis = 6378.136 km.
[e]Referenced to an ellipsoid with semimajor axis = 6378.115 km.

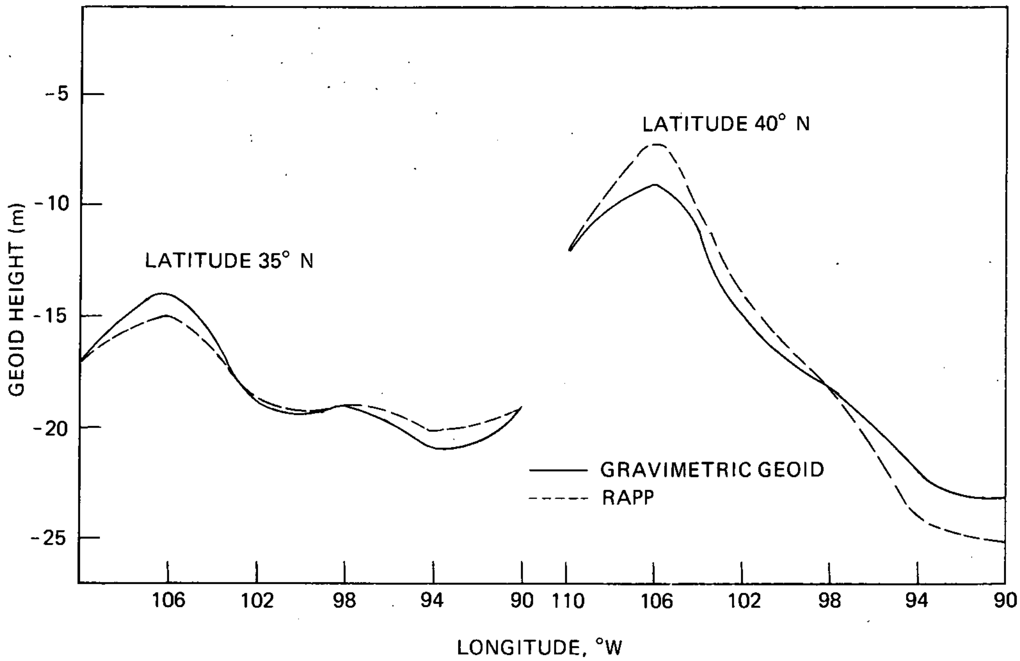

Figure 2—Detailed gravimetric geoid and western U.S. geoid of Rapp (1967).

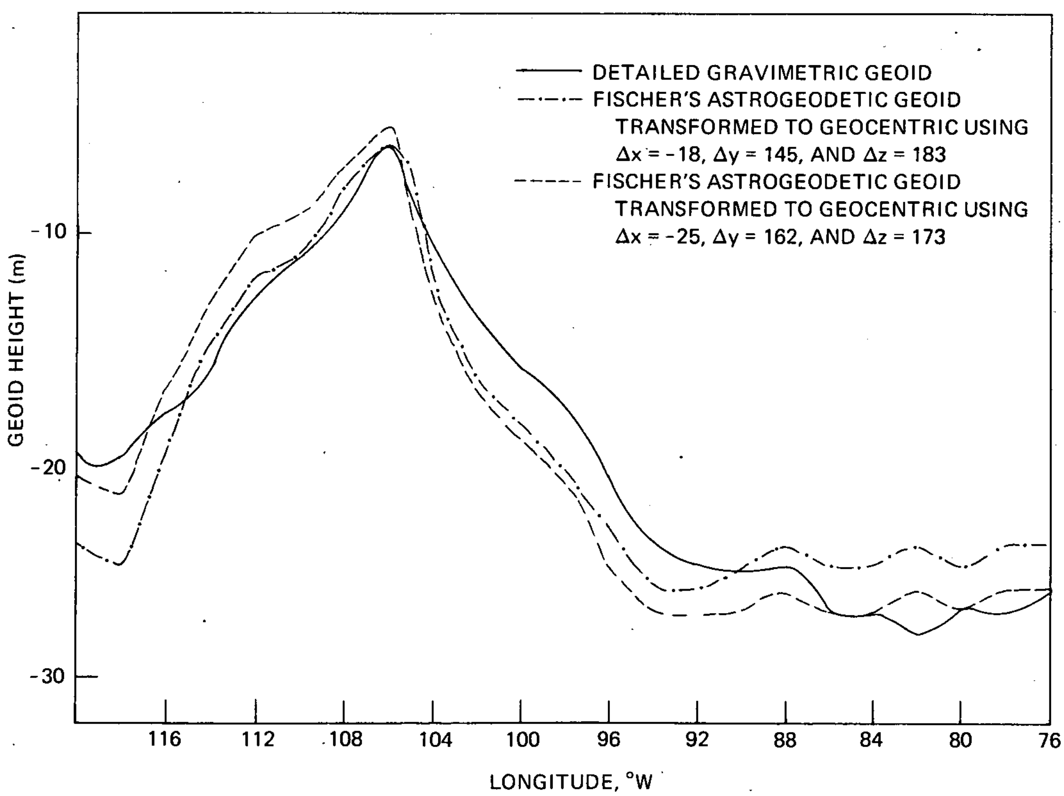

Figure 3—Detailed gravimetric geoid and Fischer's (1967) transformed astrogeodetic data at 40° N latitude.

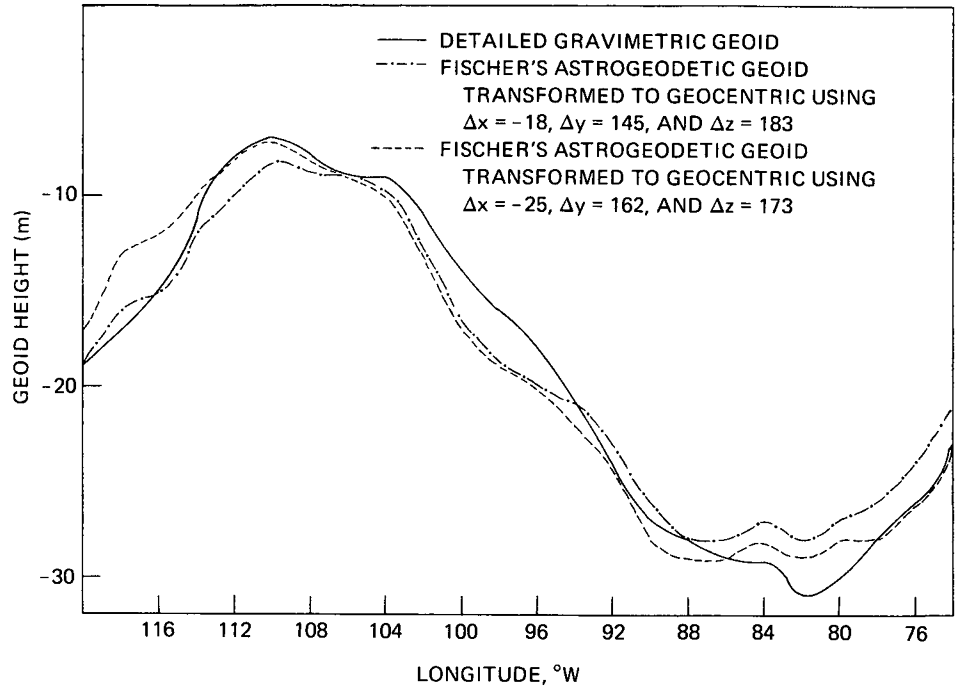

Figure 4—Detailed gravimetric geoid and Fischer's (1967) transformed astrogeodetic data at 45° N latitude.

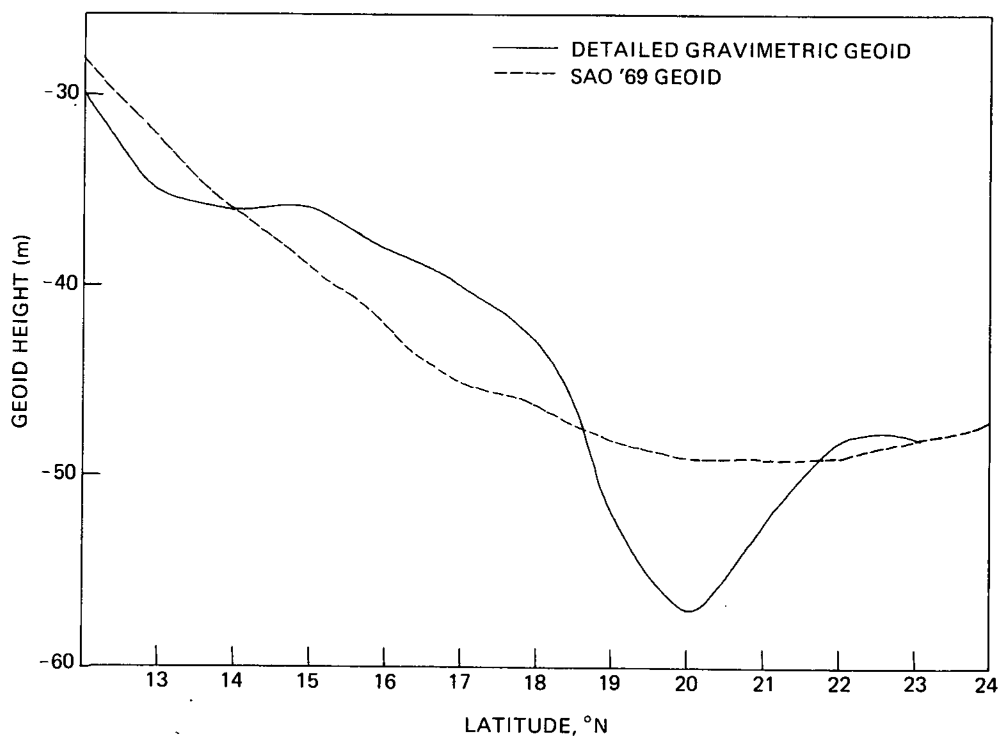

Figure 5—Detailed gravimetric geoid and SAO '69 geoid (Gaposchkin and Lambeck, 1970) at 67° W longitude.

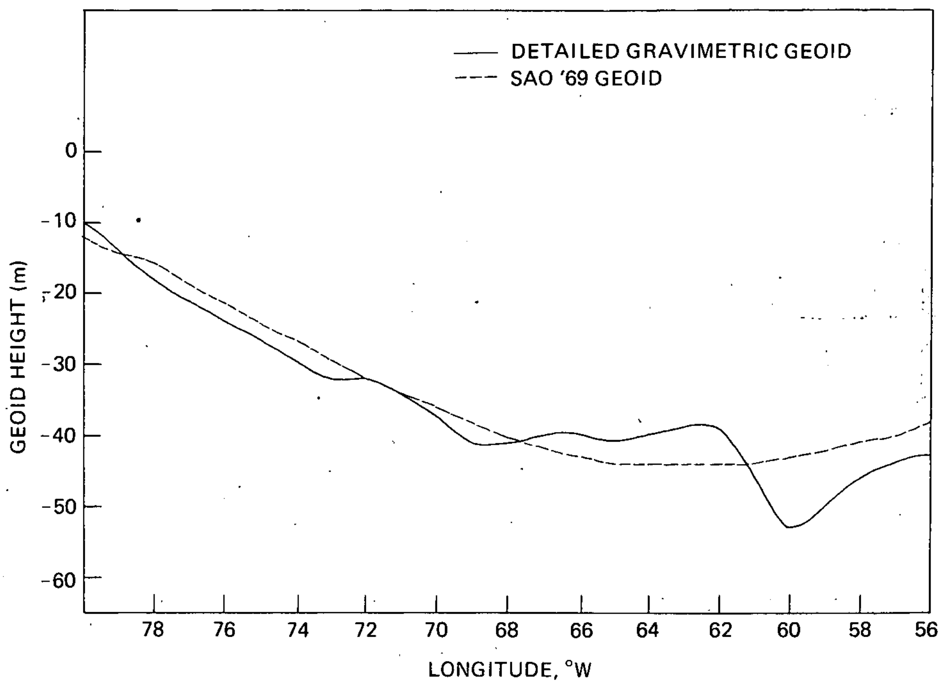

Figure 6—Detailed gravimetric geoid and SAO '69 geoid (Gaposchkin and Lambeck, 1970) at 17° N latitude.

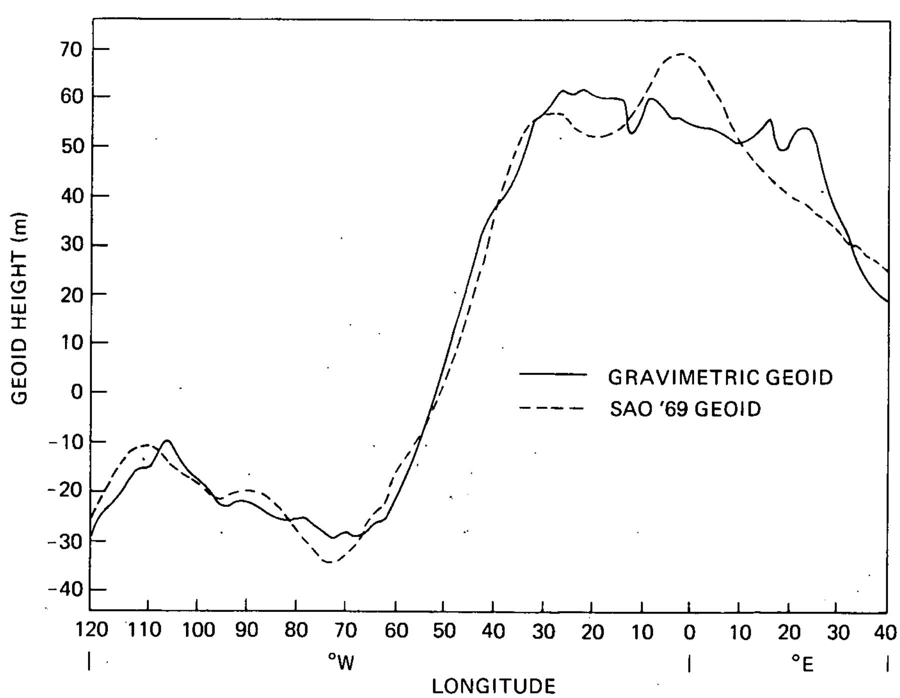

Figure 7—Detailed gravimetric geoid and SAO '69 geoid (Gaposchkin and Lambeck, 1970) at 38° N latitude.

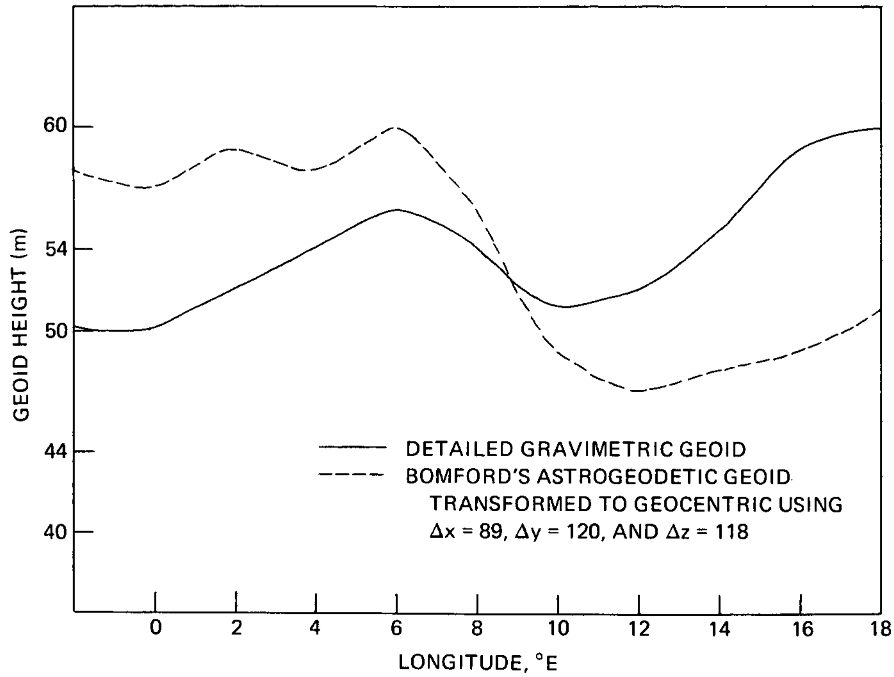

Figure 8—Detailed gravimetric geoid and Bomford's transformed astrogeodetic data at 44° N.

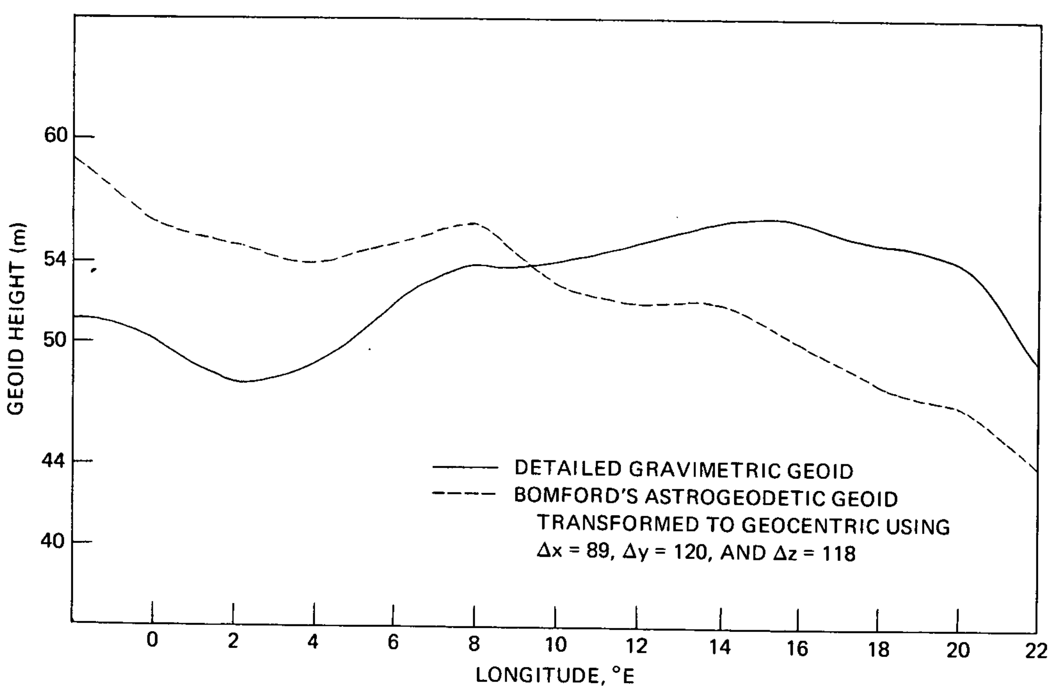

Figure 9—Detailed gravimetric geoid and Bomford's transformed astrogeodetic data at 48° N.

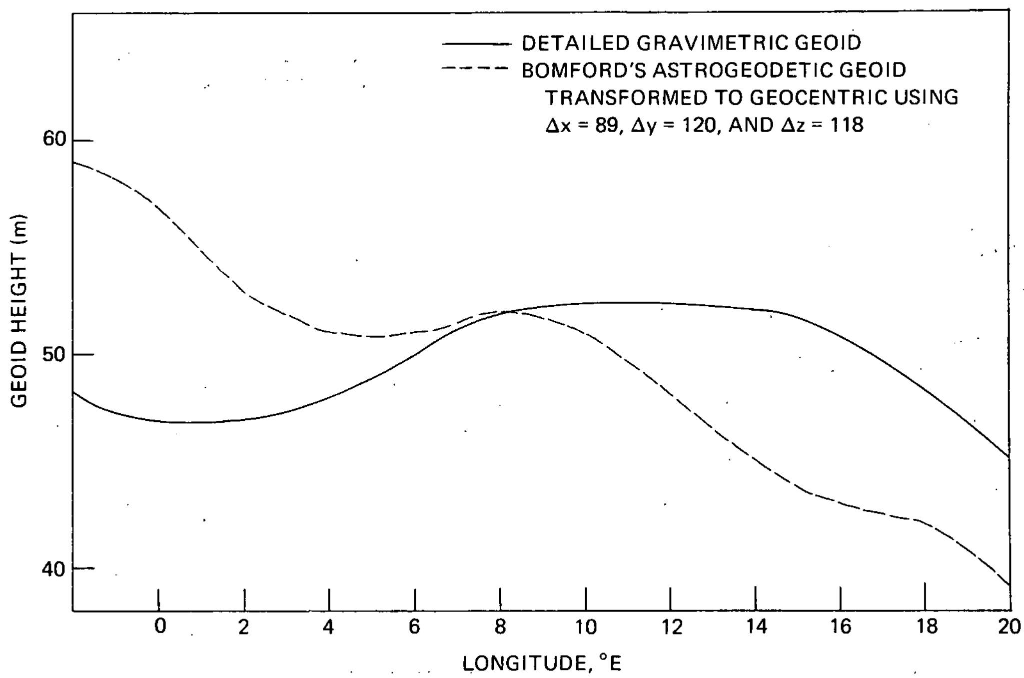

Figure 10—Detailed gravimetric geoid and Bomford's transformed astrogeodetic data at 52° N.

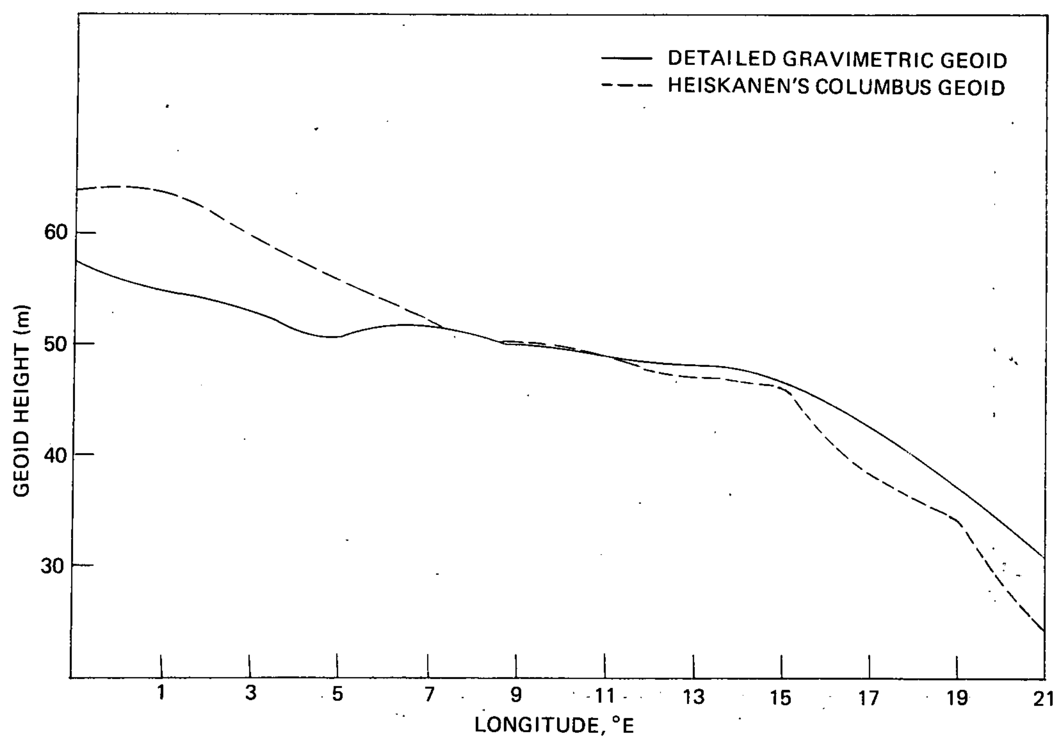

Figure 11—Detailed gravimetric geoid and Heiskanen's Columbus geoid at 35° N.

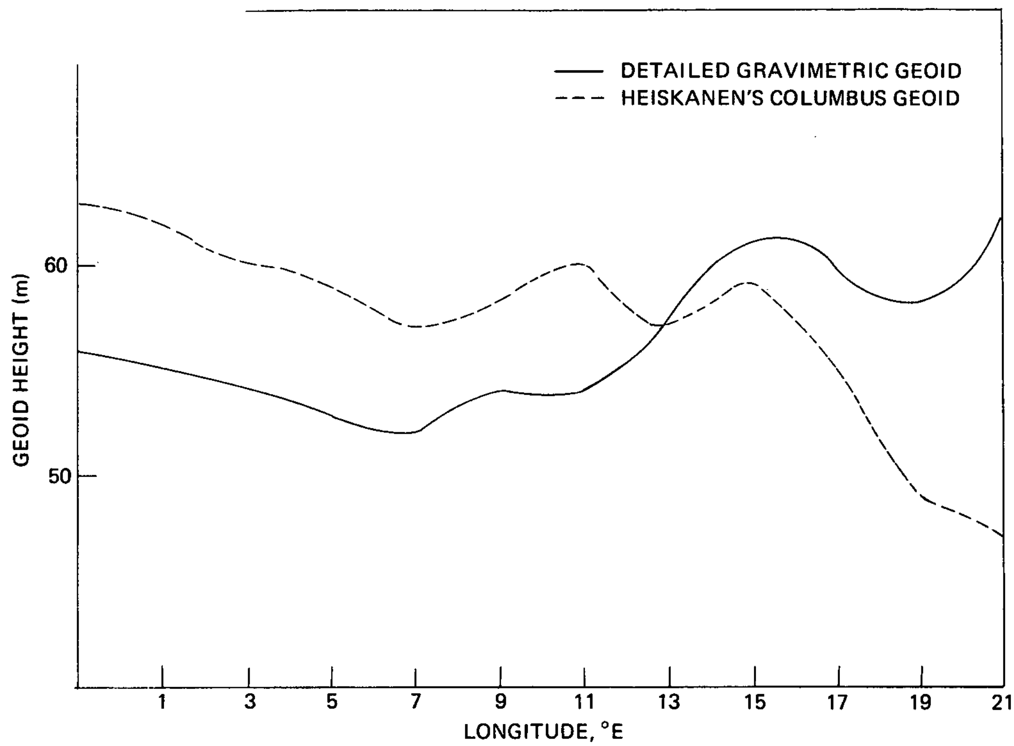

Figure 12—Detailed gravimetric geoid and Heiskanen's Columbus geoid at 40° N.

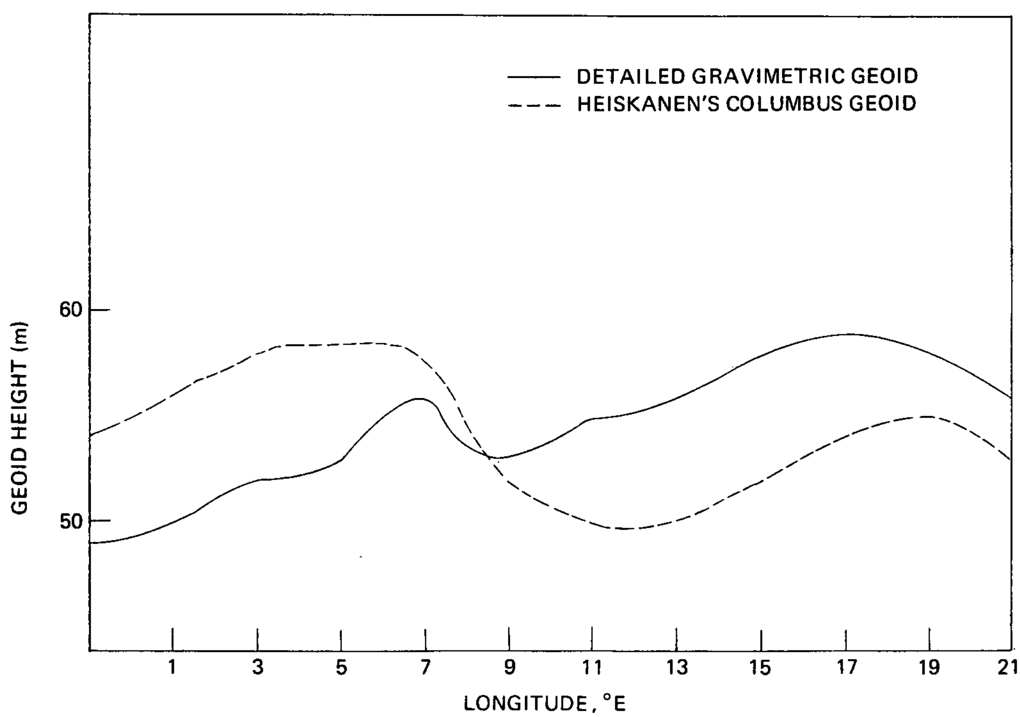

Figure 13—Detailed gravimetric geoid and Heiskanen's Columbus geoid at 45° N.

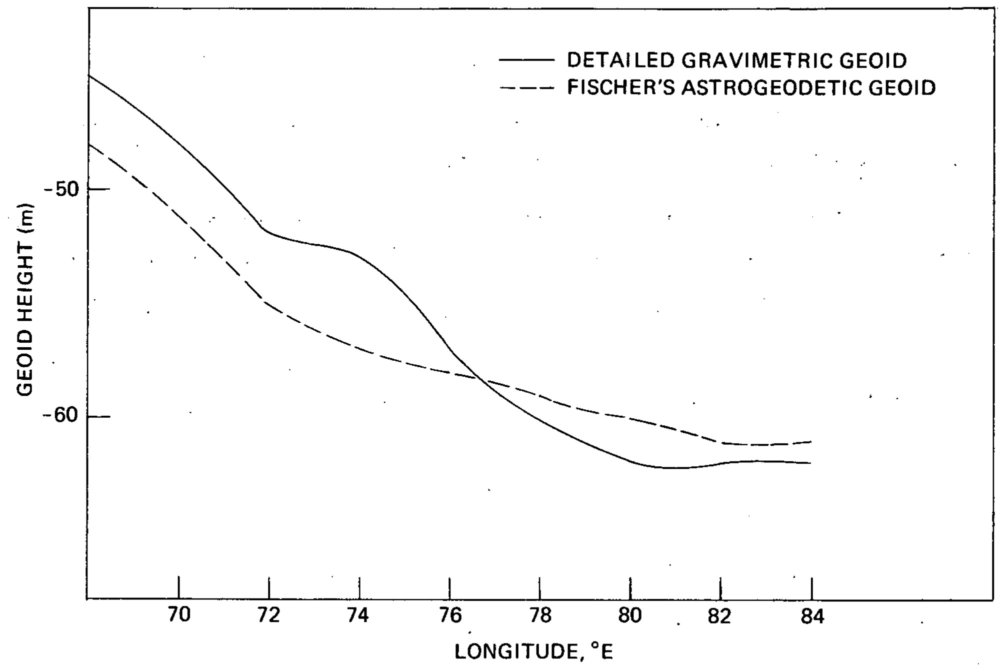

Figure 14—Detailed gravimetric geoid and Fischer's astrogeodetic geoid at 24° N.

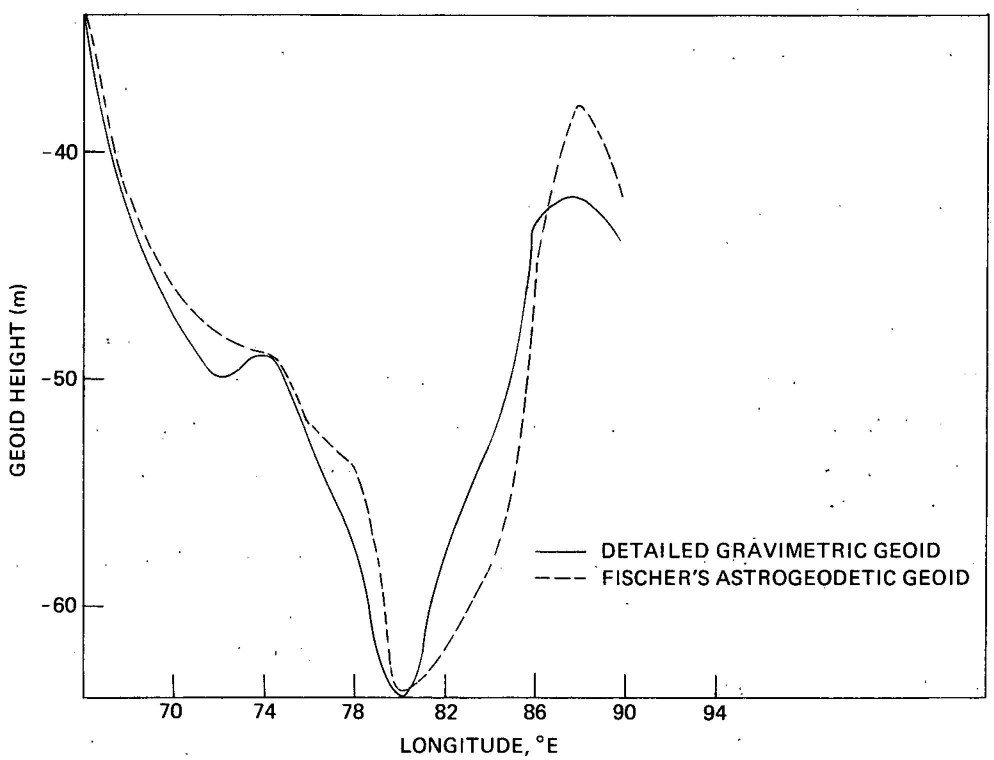

Figure 15—Detailed gravimetric geoid and Fischer's astrogeodetic geoid at 28° N.

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Appendix

Station Locations

Station Number	Station Location
2	Austin, Tex.
103	Las Cruces, N. Mex.
111	Howard County, Md.
200	Point Mugu, Calif.
400	Winter Harbor, Maine
710	Fort Wayne, Ind.
711	Stillwater, Okla.
720	Point Arguello, Calif.
734	Homestead AFB, Fla.
735	Hunter AFB, Ga.
736	Semmes, Ala.
737	Goldstone, Calif.
738	Moses Lake, Wash.
741	Organ Pass, N. Mex.
742	Beltsville, Md.
745	Stoneville, Miss.
748	Grand Forks, N. Dak.
1021	Blossom Point, Md.
1022	Fort Myers, Fla.
1030	Goldstone, Calif.
1034	East Grand Forks, Minn.
1035	Winkfield, United Kingdom
1042	Rosman, N.C.
3106	Antigua, West Indies
3402	Semmes, Ala.
3405	Grand Turk, Bahamas
3647	Dauphin Island, Ala.
3648	Hunter AFB, Ga.
3657	Aberdeen, Md.
3861	Homestead, Fla.
4061	Antigua, West Indies
4081	Grand Turk, Bahamas
4082	Merritt Island, Fla.

Station Number	Station Location
4280	Vandenberg AFB, Calif.
4860	Wallops Island, Va.
5001	Herndon, Va.
5333	Stoneville, Miss.
5861	Homestead AFB, Fla.
7036	Edinburg, Tex.
7037	Columbia, Mo.
7039	Bermuda
7040	San Juan, Puerto Rico
7045	Denver, Colo.
7050	Greenbelt, Md.
7051	Rosman, N.C.
7072	Jupiter, Fla.
7075	Sudbury, Ontario, Canada
7076	Kingston, Jamaica
8009	Delft, Netherlands
8010	Zimmerwald, Switzerland
8015	Haute Provence, France
8019	Nice, France
9001	Organ Pass, N. Mex.
9004	San Fernando, Spain
9010	Jupiter, Fla.
9021	Mount Hopkins, Ariz.
9050	Harvard, Mass.
9065	Delft, Netherlands
9066	Zimmerwald, Switzerland
9080	Malvern, United Kingdom
9091	Dionysos, Greece
9113	Edwards AFB, Calif.
9115	Oslo, Norway
9431	Riga, U.S.S.R.
9432	Uzhgorod, U.S.S.R.